Teaching mathematics in primary schools with challenging tasks: The big (not so) friendly giant

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The use of enabling and extending prompts allows tasks to be both accessible and challenging within a classroom. This article provides an example of how to use enabling and extending prompts effectively when employing a challenging task in Year 2.

This article offers a brief overview of teaching mathematics using challenging tasks. It elaborates on a particular task, which involves exploring halving as a mathematical rule in an engaging, fairytale style context. The main challenge is particularly appropriate for Year 2 students, and the article includes several actual work samples from students to illustrate some of the ways in which students may engage with the task. The relevant resources for implementing the task in a classroom are provided at the end of the article.

What are challenging tasks?

This discussion outlining challenging tasks draws heavily on a short article which appeared in *Prime Number*, published by the Mathematical Association of Victoria (Russo, 2015b).

Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, where the whole class works on the same problem (Sullivan & Mornane, 2013). The task is differentiated through the use of enabling and extending prompts. Teaching with challenging tasks enables all students to work on a similar core task, and therefore encourages them to engage with, and contribute to, the subsequent discussion around the relevant mathematics. Consequently, challenging tasks provide an appropriate means of inclusively differentiating mathematical instruction.

How do you structure a lesson using challenging tasks?

Generally teaching with challenging tasks involves a three-stage process: launch, explore, discuss (and summarise) (Stein, Engle, Smith, & Hughes, 2008).

The teacher begins by launching the challenge, which involves presenting the problem, engaging students in the relevant mathematical mindset, and highlighting resources students have at their disposal (e.g., enabling prompts, concrete mathematical materials). After the challenge is launched, students explore the task, either individually or collaboratively, and the teacher encourages students to develop at least one potentially appropriate solutions. The next stage of the lesson involves the teacher facilitating a whole-group discussion, which provides students with an opportunity to present their particular approach to solving the task.

As highlighted by Stein et al. (2008), this discussion component generally involves the teacher organising student responses in increasing order of mathematical sophistication. This sequential structure supports meaningful student participation and helps to build on the discussion of key mathematical concepts. However, the authors note that effectively coordinating this discussion can require considerable practice, skill and planning. Consequently, teachers beginning to experiment with challenging tasks in their classrooms should view it as a learning opportunity and not be overly self-critical or immediately discouraged if the discussion does not flow in the manner they expect. The teacher will usually close the lesson by offering a brief summary, reiterating the learning objective(s) and presenting a sample of student work which supports this objective.

What are enabling and extending prompts?

Enabling prompts are an integral aspect of challenging tasks. They are designed to reduce the level of

challenge through: simplifying the problem, changing how the problem is represented, helping the student connect the problem to prior learning and/or removing a step in the problem (Sullivan, Mousley, & Zevenbergen, 2006). When developing enabling prompts, it is critical that they do not undermine the primary learning objective of the lesson by 'giving too much away'. By contrast, enabling prompts may modify, and even remove, secondary learning objectives, in order to allow students who find the initial task too complex to focus on the primary learning objective (Russo, 2015a).

When engaged in a challenging task, students should be encouraged to access enabling prompts at their own initiative. Enabling prompts should be a student's first point of call if they feel they need some assistance to make progress with the problem (i.e., rather than immediately asking for support from the teacher/a fellow student). Consequently, students need explicit support around how to most effectively use enabling prompts and when to use them, particularly if they have not previously been exposed to challenging tasks. It is important, therefore, that the teacher ensures that all students know where the enabling prompts are in the room, and that there is no stigma associated with accessing an enabling prompt (e.g., an overly competitive classroom climate, where it is implicitly or explicitly assumed that 'good mathematicians don't need help').

By contrast, extending prompts are designed for students who finish the main challenge, and expose students to an additional task that is more challenging, however requires them to use similar mathematical reasoning, conceptualisations and representations as the main task (Sullivan et al., 2006).

In my classroom, I call the enabling prompts the 'hint sheet', and print one prompt on each side of this sheet. During each challenging task, I include a pile of hint sheets up the front of the classroom on a chair, so students know exactly where they are. By contrast, I call the extending prompt the 'super challenge' and generally place the extending prompt on the flip-side of the challenging task.

Why teach with challenging tasks?

Exploring mathematical ideas prior to teacher instruction supports students with their ability to reason mathematically and think critically (Marshall & Horton, 2011; Sullivan & Davidson, 2014). It has also been argued that challenging tasks, through the use of prompts, can optimise the level of challenge for a given learner (Russo, 2015a).

In addition, building a lesson around students first tackling a cognitively demanding task may also improve student persistence, as students work through the "zone of confusion" (Sullivan et al., 2014, p. 11). The zone of confusion captures the idea that being temporarily unsure how to proceed when engaged in a task is part of the process of doing mathematics. Teachers can facilitate student persistence through normalising the concept of the zone of confusion in the mathematics classroom. Students should be encouraged to view this state as a prompt for constructive action (e.g., pursuing a trial and error problem solving strategy, accessing the enabling prompt), rather than as a sign of 'failure'.

Challenging task: The big (not so) friendly giant

This task has been developed primarily with Year 2 students in mind, although I first used it in a Year 1/2 composite class (it was admittedly highly challenging for many of the Year 1 students). I have also used the challenging task in a lesson with Year 3/4 students, where the emphasis shifted to connecting the main task to the extending prompt. I would encourage teachers who are to use this task to substitute the name of the relevant town/ city describing the geographical location of their school locality, in order to make the task more enjoyable (and terrifying!) for students.

The inspiration for the task is the popular children's story *The BFG* by Roald Dahl. This story involves a number of giants, who, with one very notable exception (the big friendly giant), enjoy feasting on people —particularly children! When I first introduced this task, most students in my class were familiar with the story, which gave some comical context to the petrifying prospect of being eaten alive!

Learning objectives

The initial challenging task, as it is currently presented to students, has three learning objectives.

Primary objective

A. For students to understand that halving is a rule that makes collections (and number patterns) shrink quickly at the beginning, and more slowly as the pattern continues.

This primary learning objective connects to the *Australian Curriculum: Mathematics* content descriptions involving 'describing, continuing and creating

number patterns' outlined for Year 3 and Year 4 students. However, the emphasis on physical objects (i.e., 'collections'), as well as numbers, implies that this learning objective is also appropriate for younger students. For example, Year 1 students "investigate and describe number patterns formed by skip-counting and patterns with objects" (ACARA, 2015).

Despite these linkages, it needs to be noted that this remains a clearly ambitious learning objective, given that the concept of 'repeated halving', essentially exponential decay, does not appear formally in the *Australian Curriculum: Mathematics* until Year 9 (ACARA, 2015). However, given that a lack of understanding of exponential growth patterns has wide-ranging implications for higher level mathematics and financial literacy (Connolly & Nicol, 2015), it can be argued that students should be exposed to such ideas at a younger stage in their mathematical development. The current challenging task aims to do so in a playful, engaging context, using a tangible representation.

Secondary objectives

B. For students to explore halving patterns involving numbers greater than 20 using abstract or quasi-abstract (i.e., pictorial) representations.

C. For students to independently make the connection between the task and halving patterns.

Although having students achieve the secondary learning objectives is desirable, the focus is on the primary learning objective. Consequently, the current problem has been designed in such a manner that access to the enabling prompts suspends these secondary learning objectives.

Specifically, accessing enabling prompt 1 suspends learning objective B (i.e., by restricting the problem to numbers less than 20), whilst enabling prompt 2 suspends learning objective C (i.e., by explicitly linking the task to halving patterns).

By contrast, accessing the extending prompt introduces an additional secondary learning objective:

D. For students to make connections between related number patterns through identifying transformations.

Challenging task

A not so friendly giant moved into your street. His favourite food was Year 2 children! Actually, he refused to eat anything else!

He decided that every night, while the town slept, he was going to stick his ginormous tongue through the windows of houses and eat half of the Year 2 children in your town.

When the not so friendly giant arrived on Monday, there were 64 Year 2 children in town. How long will it take until there is only 1 Year 2 child left?

Figure 1. The Child Eater of Bern.
(Creative Commons CC-BY-SA-2.5)

To help you, have a go at completing the table (see Table 1 at the end of this article). Remember, each night the giant will eat half of the children.

Enabling prompt 1: Easier problem

When the not-so-friendly giant arrived on Monday, there were 16 Year 2 children in the town. How long will it take until there is only 1 Year 2 child left?

To help you, have a go at completing the table (see Table 3 at the end of this article). Remember, each night the giant will eat half of the children.

Enabling prompt 2: Connection to relevant prior knowledge

What patterns do you notice in these pictures?
See 'Enablng prompt 2—hint about pattern' at the end of this article for the actual pictures.

Extending prompt

Next stop for the big but not so friendly giant was the city. When it arrived in the city on Monday, there were 640 Year 2 children.

How long will it take until there are only 10 Year 2 children left in in the city? Do you notice any patterns when you compare this table to the original problem?

If the giant keeps eating children, on what day will there be approximately one child left in the city? See the template for the extending prompt at the end of this article — Table 2.

Relevant questions for the post-task discussion

During the post-class discussion, students should be encouraged to describe their various approaches to the task(s) and the conclusions they reached. Relevant questions for encouraging student reflection and reinforcing the learning objectives include:

- When looking at your table, what are the main things you notice?
- Why do you think it is that the giant is eating a lot of children at the start of the week, and fewer and fewer children as the week goes on—even though he is always following the same rule (that is, he is always eating half the children each day)? What does this tell us about halving patterns?
- If the giant kept following the halving rule and was allowed to eat fractions of children, do you think the giant would ever run out of children to eat? Why? Why not?
- What do you think was the most efficient way of approaching the challenge?
- Having listened to other students' approaches to the challenge, how would you tackle this task differently next time?

Student work samples

I have included four student work samples, taken from Year 2 students, outlining various student attempts at the big (not so) friendly giant challenge. I briefly describe how each of these students made progress with the task below.

Jack's attempt at the challenge

Figure 2. Jack's attempt.

Jack was highly distracted at the beginning of the 'explore stage' of the lesson, a clear indication that he thought the challenging task too difficult. Jack struggles to manipulate numbers larger than 20, and certainly does not yet have fluent knowledge of his doubles (and halves) facts. Jack was encouraged to get himself the 'hint sheet', and explicitly directed to study Enabling Prompt 2: Connection to relevant prior knowledge. In particular, Jack was asked to see if he could notice any patterns that he might recognise. After some time, Jack appeared to recognise the halving pattern, noting that "half the pink elephants are disappearing". Jack was encouraged to have a go at the easier problem (Enabling Prompt 1), and told that "every day, half the children are disappearing into the giant's stomach, just like the elephants". Armed with this new perspective on the problem, Jack was able to use the pictorial representation in the easier problem to make progress with the challenge. Although Jack did not achieve any of the secondary learning objectives outlined, he was still able to engage with the primary learning objective; specifically, that the process of repeated halving rapidly reduces the number of objects in a collection.

John's attempt at the challenge

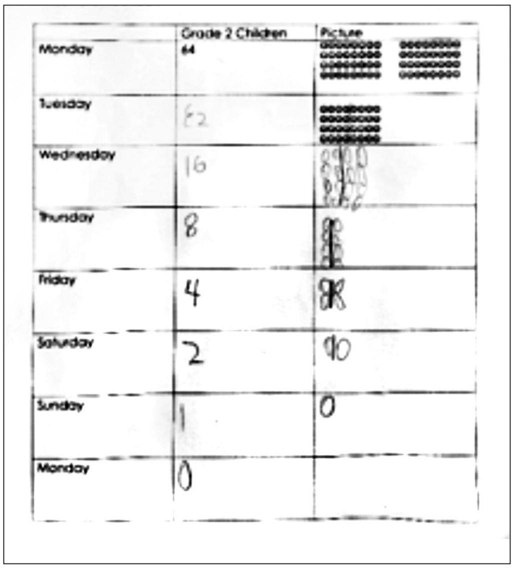

Figure 3. John's attempt.

This attempt at the challenging task by John relied on repeatedly halving the pictorial representation, and then counting how many children were left on a particular day. In contrast to Jack, John demonstrated a conceptual understanding of halving without first having to study the enabling prompt. This allowed him to be highly systematic in the manner in which

he approached the task. However, John's reliance on the pictorial representation at all stages of the problem implied that he did not yet possess procedural fluency in relation to his doubles and halves facts, and/or that he found it difficult to move between the pictorial and the numerical representation of the problem.

Like many other students, John's apparent conceptual understanding of halving did not generalise from whole number concepts to fractional concepts. Specifically, John assumed that were the pattern to continue, halving 1 would equal 0. This provided an opportunity for me to attempt to link whole number thinking to fractional thinking, by reframing the question as: "What is half of one whole?". I also encouraged John to explore the halving pattern in reverse, focussing instead on doubling. This lead to provocative questions intended to challenge John's misconception, such as: "Does double zero equal one? Can you prove it?".

Ella's attempt at the challenge

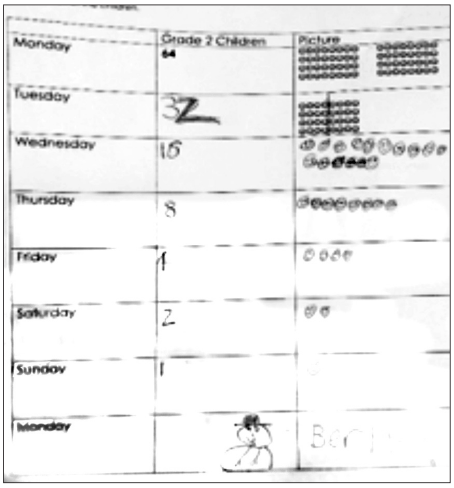

Figure 4. Ella's attempt.

Although the above work sample appears similar to John's attempt at the task, a deeper analysis is revealing. This work sample is from Ella, who also relied on pictorial representations, in order to systematically halve 64 and then halve 32. However, having worked out that there were 16 children left on Wednesday, she shifted to relying on her pre-existing knowledge of double (and half) facts to 20, performing a series of mental calculations to finish the table. She subsequently drew the additional faces to, in her words, "finish the problem". Consequently, Ella effectively demonstrated a conceptual understanding of halving, as well as the relevant procedural fluency. Most interestingly, and in contrast to John, she was able to shift her approach halfway through the problem, to ensure that she relied on mental strategies whenever possible. In her own words: "Whenever I can, I want to do it in my head".

Harvey's attempt at the challenge

Figure 5. Harvey's attempt.

In contrast to the other three students, who relied at least in part on the pictorial representation, Harvey rapidly solved the initial challenge mentally, describing his use of a "number splitting" strategy. Specifically, Harvey described how he broke up the number into its place value components (e.g., 30 and 2), and then halved each component separately (e.g., 15 and 1), and then recombined the two components to arrive at the total (e.g., 16). Harvey was able to apply this same process to successfully attempt the extending prompt, although he did not immediately make the connection between this and the main challenge.

After completing the extending prompt, Harvey was asked to systematically compare it to the main challenge. Following some consideration, this led to the comment that: "they are the same except this one (gesturing to the extending prompt) has a zero at the end of it". I left Harvey to contemplate what this might mean, and, pointing to the main challenge, asked him: "What if there had of been 64,000 children on Monday? (writing the number 64,000 for him). Could you work out how many children would be there on the Sunday, without doing any calculations?".

Conclusion

In my experience, there is enormous potential for teaching with challenging tasks, even in the early years of schooling when students ostensibly have relatively little 'formal' mathematical knowledge. One of the strengths of challenging tasks is the manner in which they can be used to differentiate instruction through the use of enabling and extending prompts, allowing

students at different levels to pursue the same primary learning objective. For example, in our Year 2 class, both Jack and Harvey developed some appreciation for halving patterns, despite the former thinking mathematically at (probably) a Year 1 level, and the latter at (probably) a Year 4 level.

Although teaching with challenging tasks involves careful planning beforehand (in particular, the designing of the prompts), I have found the actual experience of teaching with challenging tasks far from an orderly process. It is simultaneously stimulating, engaging, chaotic, frustrating and fun; and, ultimately, I think, very effective for deepening student conceptual understanding and encouraging flexible and creative mathematical thinking.

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The big (not so) friendly giant: Main challenge

When the not so friendly giant arrived on Monday, there were 64 Year 2 children in the town. How long will it take until there is only one Year 2 child left? To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

Table 1. Main challenge.

	Year 2 children	Picture
Monday	64	☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Tuesday		☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺ ☺☺☺☺☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

The big (not so) friendly giant: Extending prompt

Next stop for the big but not so friendly giant was the city. When it arrived in the city on Monday, there were 640 Year 2 children in the city. How long will it take until there are only 10 Year 2 children left? Do you notice any patterns when you compare this table to the original problem? If the giant keeps eating children, on what day will there be approximately one child left?

Table 2. Extending prompt.

	Year 2 Children
Monday	640
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	
Monday	

The big (not so) friendly giant: Enabling prompt 1—easier problem

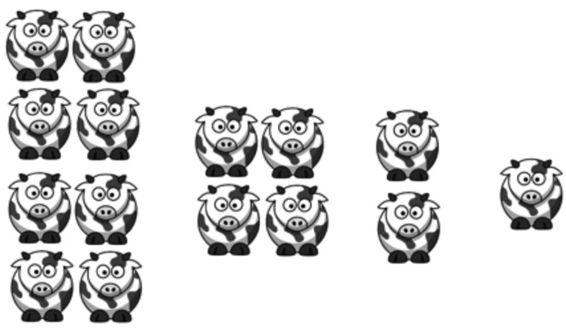
When the not so friendly giant arrived on Monday, there were 16 Year 2 children in your town. How long will it take until there is only one Year 2 child left? To help you, have a go at completing the following table. Remember, each night the giant will eat half of the children.

Table 3. Enabling prompt 1—easier problem.

	Year 2 children	Picture
Monday	16	☺☺☺☺ ☺☺☺☺ ☺☺☺☺ ☺☺☺☺
Tuesday		☺☺☺☺ ☺☺☺☺
Wednesday		
Thursday		
Friday		
Saturday		
Sunday		
Monday		

The big (not so) friendly giant: Enabling prompt 2—hint about pattern

What patterns do you notice in this picture?



What patterns do you notice in this picture?